

Solar Neutrino Oscillations in the Moon

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Many different solar neutrino experiments detect significantly fewer neutrinos than expected. No known modification of the standard solar model can account for all of the data. The most likely explanation is neutrino oscillations, in which many of the electron neutrinos emitted by the core of the Sun convert into other types of neutrinos while traversing the Sun. This explanation will be severely tested within the next few years. We present a pedagogical explanation of the phenomenon of neutrino oscillations, and then use the results to address the question of whether the currently planned neutrino detectors could detect neutrino oscillations in the moon during solar eclipses.

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1 Introduction

A quarter of a century ago, Davis, Harmer and Hoffman [1] reported the results of the Homestake solar neutrino experiment. They found that the flux of neutrinos emitted by the Sun was a factor of $1/4$ to $1/3$ of the expected flux. Their result was confirmed several years ago by experimenters at Kamiokande, Japan[2]. It implies that either (a) the Standard Solar Model (SSM), used to predict the neutrino flux, is wrong or else that (b) the neutrinos *are* emitted, but changed into an undetectable type of neutrino somewhere between the center of the Sun and the detector.

In the past few years, the experiments of SAGE[3] and GALLEX[4] have also reported significant deficits in the solar neutrino flux. Since the neutrinos detected in these experiments come mostly from the primary energy production mechanism in the Sun, there are much smaller uncertainties in the Standard Solar Model prediction, implying that the solution to the solar neutrino problem lies in the neutrino physics (case (b) above) rather than the SSM (case (a)).

How can neutrinos “change” into an undetectable type of neutrino? As will be discussed in detail below, there are three types of neutrinos, referred to as “flavors”: electron neutrinos, muon neutrinos and tau neutrinos. Only electron neutrinos can be emitted by the Sun, and only those can be detected in solar neutrino experiments. However, if neutrinos have mass, then the mass eigenstates could be linear combinations of the three types of neutrinos, and one can easily show that electron neutrinos could transform into one of the other two types during travel from the core of the Sun to the detector on Earth, and thus evade detection. The preferred oscillation mechanism, called the Mikheyev, Smirnov, Wolfenstein (MSW) mechanism[5, 6], gives an excellent description of the data for all four solar neutrino experiments. In this mechanism, neutrino oscillations in matter differ from those in vacuum, and can be resonantly enhanced when the

neutrinos encounter particular densities; the solar neutrinos then encounter these densities on their way out of the Sun and, possibly, also on their way through the Earth (at night).

Many particle physicists believe that the most exciting results in particle physics during the next few years will come from neutrino experiments. The Solar Neutrino Observatory in Sudbury, Canada, will begin operation next year and will be the first experiment which is sensitive to muon and tau neutrinos (detection of such neutrinos, since they cannot be produced in the Sun, would be a “smoking gun” for neutrino oscillations). The CHORUS and NOMAD experiments, which are beginning to take data now, will be sensitive to the theoretically preferred values for muon-tau neutrino oscillations. Super-Kamiokande will begin operation within two years, and there are already various hints of neutrino oscillations from atmospheric neutrino measurements. Confirmation of neutrino oscillations would be of enormous importance to particle physics, and would be the first evidence of physics outside the standard model in two decades.

Our purpose in this article is twofold. First, the basic physics of neutrino oscillations does not require any more understanding than junior level quantum mechanics, and we hope to provide an explanation of neutrino oscillations which will enable an undergraduate to follow the exciting imminent developments in the field. Secondly, we wish to propose and analyze the remarkable possibility that future neutrino experiments might be able to detect changes in the solar neutrino flux during a solar eclipse, due to resonant MSW oscillations in the moon. In Section II, we will provide some general background, as well as a discussion of the experiments used to measure the solar neutrino flux. Sections III and IV will deal, respectively, with the theory of neutrino oscillations in vacuum and with the resonant enhancement in matter. In Section V, the effects of a non-constant density medium will be analyzed. The calculation of the influence of the moon on the solar neutrino flux will be discussed in Section VI. Finally, Section VII will contain our

conclusions and a brief discussion of the future prospects of such an experiment.

2 Background

The existence of the neutrino was first postulated by Pauli in 1931 to explain the continuous energy spectrum of electrons in beta decay. Direct evidence for neutrinos, however, did not materialize until 1953 when Reines and Cowan exposed a sample of hydrogen in an intense flux of antineutrinos from a nuclear reactor and observed the consequent stimulated beta decay of the protons.

Today, we know that neutrinos are electrically neutral, possibly massless, spin 1/2 particles which interact very weakly with ordinary matter. A typical solar neutrino will have a mean free path in lead of approximately a light year, and thus neutrino detectors must look at a large flux of neutrinos in a very large detector, in order to have a reasonable probability of observing a reaction. There are three different types of neutrinos, corresponding to the three generations of massive leptons—the electron, the muon and the tau. Interactions involving the electron neutrino, for example, only involve the electron. Since the temperature of the Sun is not high enough to produce muons or taus, only electron neutrinos are emitted by the core of the Sun.

As soon as it became clear that the primary energy production mechanism in the Sun was nuclear fusion, it was recognized that neutrinos would be emitted in the process. There are several reaction chains which convert hydrogen into helium in the core of the Sun. The proton-proton chain, which accounts for most of the energy production, produces very low energy neutrinos. Other chains produce different neutrino spectra; the highest energy neutrinos arise from the conversion of ${}^7\text{Be}$ to ${}^8\text{B}$ in the boron cycle (which only accounts for about 10^{-4} of the energy production). Predictions of the solar neutrino flux are generated from a complicated numerical simulation of the Sun known as the

standard solar model (SSM). The model itself is derived from some fairly simple assumptions (such as hydrostatic equilibrium), although the computer codes used to generate the predictions can be quite complicated. The current SSM is a dynamical theory that agrees with many known facts about the Sun, such as spectral data, helioseismological data, and its current mass, radius and luminosity. The model predicts not only the flux of solar neutrinos, but the energy spectrum as well.

Two processes are used to observe the solar neutrino flux: neutrino absorption and neutrino-electron scattering. The neutrino absorption experiments all use some form of the stimulated beta decay reaction $\nu_e + {}^A_Z \rightarrow {}^A_{(Z+1)} + e^-$, where Z is the charge of the nucleus and A is the mass number. Note that only electron neutrinos can be detected in this manner (the energy of solar neutrinos is in the MeV range, too small to produce muons or taus). The solar neutrino experiment of Davis, et al., at Homestake, South Dakota, is based on the reaction $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$. Since the energy of the ${}^{37}\text{Ar}$ is higher than that of the ${}^{37}\text{Cl}$, only sufficiently high energy neutrinos can initiate this reaction. It turns out that the threshold for this reaction is so high that only the high energy neutrinos from the rare boron cycle can be detected. The SAGE and GALLEX experiments use the conversion of gallium into germanium. This reaction has a much lower threshold, and thus these experiments are sensitive to the neutrinos coming from the primary proton-proton chain. In all of these experiments, the final nuclear product is radioactive. The apparatus is allowed to stand for a long period of time—several months for the Homestake experiment—and then the radioactive atoms are chemically separated out from the detector and counted in proportional counters. Thus, they have a time resolution on the order of months (and would obviously be useless for looking for a change in the flux during an hour-long solar eclipse).

One exception will be the Sudbury Neutrino Observatory (SNO) which will observe radiation from the produced electrons in the reaction $\nu_e + d \rightarrow p + p + e^-$ and thus obtain

real-time measurements of the electron neutrino flux, direction and energy spectrum. It is expected to have a very high count rate (on the order of one event per hour).

The second type of reaction used to observe solar neutrinos is neutrino-electron scattering, or $\nu + e \rightarrow \nu + e'$. The Japanese experiment at Kamiokande uses this reaction; SNO will also use this reaction in addition to the above absorption reaction. Typically, the detector material is water. Phototubes are used to detect the radiation from the scattered electrons. The rate for this reaction is lower than for neutrino absorption; an upgrade of Kamiokande should have several events per day.

The Homestake experiment detects 1/4 to 1/3 the expected flux; Kamiokande sees a similar deficit. The SAGE and GALLEX experiments, sensitive to a different energy range, see approximately 60 percent of the expected rate. There have been attempts to modify the standard solar model to account for these discrepancies, but these attempts have run into serious problems. For example, attempts to make the core temperature lower (which would drastically cut the neutrino production rate) also lower the luminosity too much, etc. No solar model has been successful in explaining the solar neutrino fluxes and still maintaining agreement with the observed mass, radius, luminosity and helioseismology data.

If modifying the model of the Sun can't explain the solar neutrino deficit, then one must modify the physics of neutrinos. The most attractive such modification involves neutrino oscillations. The "electron neutrino", $|\nu_e\rangle$, is *defined* to be the state which, in weak interactions, is always associated with electrons; similar definitions apply to the muon and tau neutrinos. If neutrinos are massless (or have the same mass), then there is no other way to distinguish between the three states. However, suppose neutrinos have mass. Then there will be three states $|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$ which have masses m_1, m_2, m_3 . How are the states $|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle$ related to $|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$? Each set forms a complete description of the three neutrinos. If one had a detector which made a "measurement"

of the masses, then the “mass basis”, $|\nu_1\rangle, |\nu_2\rangle, |\nu_3\rangle$, would provide a description of the possible eigenstates of the measurement; if one had a detector which measured the weak interactions of the neutrinos (such as a solar neutrino detector), then the “flavor basis”, $|\nu_e\rangle, |\nu_\mu\rangle, |\nu_\tau\rangle$, would provide a description of the eigenstates of the measurement. Since each basis forms an orthonormal set, they can be related by a unitary transformation. For example (ignoring mixing with the tau family), one would expect

$$\begin{aligned} |\nu_1\rangle &= |\nu_e\rangle \cos \theta + |\nu_\mu\rangle \sin \theta \\ |\nu_2\rangle &= -|\nu_e\rangle \sin \theta + |\nu_\mu\rangle \cos \theta \end{aligned} \tag{1}$$

where θ is an arbitrary angle which must be experimentally determined.

As we will see in the next section, if an electron neutrino is produced in the core of the Sun, then it propagates as a combination of ν_1 and ν_2 and, depending on the mass difference and mixing angle, this combination will be different when the detector is reached, leading to a combination of ν_e and ν_μ . As the latter are not detected, there will be a deficit in the neutrino flux. Not surprisingly, there will be a difference between propagation in vacuo and propagation in the Sun or Earth (or moon), since there is an effective index of refraction in a medium. The two free parameters in neutrino oscillations are Δm^2 and $\sin^2 2\theta$, where the former is the mass-squared difference between the electron neutrino and the (presumably) muon neutrino and the latter is the mixing angle. One of the major purposes of any solar neutrino experiment is to constrain the possible values of these parameters. The data from all existing solar neutrino experiments have been combined by Hata and Langacker[8], who have narrowed the values down to the region shown in Fig. 1. Future experiments will, within the next couple of years, either narrow the region further, or rule out neutrino oscillations as the solution of the solar neutrino problem. The smoking gun may come from SNO, since the neutrino-electron scattering reaction discussed above will be also sensitive (with a different electron distribution) to

muon and tau neutrinos; their detection would definitely establish neutrino oscillations as the solution, and would also establish that these fundamental particles do have a mass.

3 Neutrino Flavor Oscillations in Vacuum

The idea of neutrino flavor oscillations is straightforward: if a beam of pure electron neutrinos, for example, is generated at some point then as the beam propagates through space it will acquire some small muon neutrino component. At some point, the muon neutrino component reaches a maximum, after which the beam begins to oscillate back into a pure electron neutrino state. If the resonance condition (to be discussed later) is satisfied, then at this maximum point the beam will consist entirely of muon neutrinos. If you had a neutrino detector which you could move along the beam, you would find that the flavor content of the beam varied sinusoidally with your distance from the point at which the neutrinos originated. Many experiments use precisely such a procedure to look for neutrino oscillations directly. We will spend the rest of this section making this argument quantitative. Our discussion follows to some extent that of Bernstein and Parke[9].

We will consider the case of two neutrino flavors (the extension to three flavors is trivial, and will not be relevant in the following). There are two wavefunctions, $|\nu_1\rangle$ and $|\nu_2\rangle$ with masses m_1 and m_2 , respectively. Since the neutrinos from the Sun have energies in the MeV range, and the experimental bounds on the masses of the lightest two neutrinos are much smaller than this, the neutrinos are moving at a speed very close to the speed of light. The time evolution of these wavefunctions will be governed by the relativistic analog of the Schrödinger equation (this can be easily derived from the Dirac equation):

$$i\frac{d\nu_1}{dt} = E_1\nu_1 = \sqrt{k^2 + m_1^2} \nu_1$$

$$i\frac{d\nu_2}{dt} = E_2\nu_2 = \sqrt{k^2 + m_2^2} \nu_2 \quad (2)$$

where we use units of $\hbar = c = 1$ and drop the ket symbol. This can be written as

$$i\frac{d}{dt} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = \begin{pmatrix} \sqrt{k^2 + m_1^2} & 0 \\ 0 & \sqrt{k^2 + m_2^2} \end{pmatrix} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} \quad (3)$$

This equation can be trivially solved, yielding

$$\begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = \begin{pmatrix} \exp[-i\sqrt{k^2 + m_1^2} t] & 0 \\ 0 & \exp[-i\sqrt{k^2 + m_2^2} t] \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix} \quad (4)$$

This propagator matrix can be used to determine the wavefunctions at any time given the initial components $\nu_1(0)$ and $\nu_2(0)$.

Since the neutrinos are ultrarelativistic, we can expand the phases, using $\sqrt{k^2 + m^2} \simeq k + \frac{m^2}{2k}$. Defining the intrinsically positive quantity (without loss of generality, one can assume that $m_2 > m_1$)

$$\Delta m_o^2 = m_2^2 - m_1^2, \quad (5)$$

the phases can then be expressed as

$$\sqrt{k^2 + m_i^2} = [k + \frac{m_i^2 + m_1^2}{4k}] \mp \frac{\Delta m_o^2}{4k} \quad (6)$$

taking the minus sign for $i = 1$ and the plus sign for $i = 2$. Note, however, that the bracketed term is a common phase which can be factored out of the propagator matrix. It then acts as an overall phase factor to the entire wavefunction, and can be discarded since overall phases are not measurable. We are left with

$$\vec{\nu}(t) = \begin{pmatrix} \exp[i\frac{\Delta m_o^2}{4k}t] & 0 \\ 0 & \exp[-i\frac{\Delta m_o^2}{4k}t] \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix}, \quad (7)$$

or, equivalently,

$$i\frac{d}{dt} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m_o^2}{4k} & 0 \\ 0 & \frac{\Delta m_o^2}{4k} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \quad (8)$$

To determine the flavor transition probabilities, we need to express this result in terms of the flavor basis. Using Eq. 1 to relate ν_1 and ν_2 to ν_e and ν_μ , we find that

$$i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m_o^2}{4k} \cos 2\theta & \frac{\Delta m_o^2}{4k} \sin 2\theta \\ \frac{\Delta m_o^2}{4k} \sin 2\theta & \frac{\Delta m_o^2}{4k} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \quad (9)$$

From this, it is clear that a beam which starts as an electron neutrino beam will evolve into a beam with both electron neutrino and muon neutrino components. Solving this equation (most easily done by looking at Eq. 7 in the flavor basis) yields

$$\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = \begin{pmatrix} \cos(\frac{\Delta m_o^2}{4k}t) + i \cos 2\theta \sin(\frac{\Delta m_o^2}{4k}t) & i \sin 2\theta \sin(\frac{\Delta m_o^2}{4k}t) \\ i \sin 2\theta \sin(\frac{\Delta m_o^2}{4k}t) & \cos(\frac{\Delta m_o^2}{4k}t) - i \sin 2\theta \sin(\frac{\Delta m_o^2}{4k}t) \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_\mu(0) \end{pmatrix}. \quad (10)$$

If one starts with a beam of one type of neutrino, then the probability of conversion at a later time is then

$$P_{\text{transition}}(k, t) = |\langle \nu_\mu(t) | \nu_e(0) \rangle|^2 = \sin^2 2\theta \sin^2(\frac{\Delta m_o^2}{4k}t) \quad (11)$$

If we now convert this to a distance $L = ct$, and express Δm_o^2 in units of eV^2 , L in units of kilometers, and the energy of the neutrino in units of MeV, the expression becomes

$$P_{\text{transition}}(k, t) = \sin^2 2\theta \sin^2(1.27 \frac{\Delta m_o^2}{k} L). \quad (12)$$

There are several important features of this result. First, it was assumed that the neutrinos were emitted at a point; if the production region is in fact on the order of or greater than the oscillation wavelength, then the second sin term averages to $1/2$, leading to a probability of transition given by $\frac{1}{2} \sin^2 2\theta$, which is never bigger than $1/2$. This could not account for the signal reduction of greater than 50 percent observed at Homestake². Secondly, if Δm_o^2 is very small, so that the oscillation length is very long, then one can fine-tune so that one astronomical unit is an integral number of oscillation

²Of course, the errors in the SSM and the experiment might accommodate a 50 percent reduction; one can also mix all three neutrinos equally and get a slightly bigger reduction.

lengths (the integer can not be too large, since no significant seasonal variation is seen); this requires a $\Delta m_o^2 \simeq 10^{-10} \text{eV}^2$. Even then, a very large mixing angle is needed to get the large reduction in signal observed by Homestake; and since the oscillation length is energy dependent, it is difficult to also get agreement with the SAGE and GALLEX reductions. Thus, vacuum oscillations are not theoretically favored and can not be made consistent with all solar neutrino experiments.

As mentioned previously, the two parameters Δm_o^2 and θ are free: their values must be measured experimentally. The details of an experiment, such as the length between the neutrino source and the neutrino detector, the neutrino spectrum, and systematic uncertainties, determine some region of parameter space which may be explored. The usual procedure is to determine the isoprobability contours for a flavor transition and plot these in the Δm_o^2 vs. $\sin^2 2\theta$ plane (or MSW plane). For example, if a rate of five events per hour is predicted, and a flux of one per hour is observed, then all regions of the plane which predict anything other than an 80 percent transition probability are excluded. The resulting line is then smeared out into a region by experimental and theoretical uncertainties. In most laboratory experiments to date, no reduction is seen, so one plots limits in the MSW plane.

In the next section, we will see that the interactions of neutrinos with matter can significantly enhance flavor oscillations, leading to possibly large transition probabilities even if the vacuum mixing ($\sin \theta$) is small.

4 Neutrino Flavor Oscillations in Matter

Wolfenstein pointed out in 1978 that when a neutrino propagates through matter, it will interact via the weak force with the surrounding electrons. Both electron and muon neutrino states are capable of scattering off an electron (or a proton or neutron) by ex-

changing a Z^0 ; this is called the weak neutral current interaction. The electron neutrino, however, can additionally interact via the weak charged current by exchanging a W^+ , as shown in Fig. 2. The net effect of these interactions is to slow down the neutrinos' propagation—with the electron neutrino states being slowed down more than the muon neutrino states due to the extra mode of interaction available to them. This additional phase difference can enhance or repress flavor oscillations. A beautiful discussion of this effect can be found in a paper of Bethe[10].

In this section we will discuss the case of neutrino propagation through a medium of constant density. In the following section we will outline the extension of this theory to a medium of slowly changing density (the adiabatic case) and a medium whose density changes very rapidly (the non-adiabatic case).

First, consider the effect of the weak neutral current on the propagator matrix in Eq. 9. Since the weak neutral current acts equally on the electron and muon neutrino states, the term which appears in each diagonal element resulting from the weak neutral current Hamiltonian is the same. If a term proportional to the identity matrix is added to the Hamiltonian, then it will end up factoring out of the solution and once again contribute an overall phase to the wavefunction, thus having no effect on oscillations. Thus, the weak neutral current will not affect neutrino oscillations.

The weak charged current, on the other hand, acts *only* on electron neutrino states, so there will be an added term only in the upper left element of the Hamiltonian. This term is

$$\sqrt{2}G_F N_e \quad (13)$$

where G_F is the Fermi coupling constant which characterizes the strength of the weak interaction and N_e is the electron number density in the medium. Eq. 9 now looks like

$$i \frac{d}{dt} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m_o^2}{4k} \cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m_o^2}{4k} \sin 2\theta \\ \frac{\Delta m_o^2}{4k} \sin 2\theta & \frac{\Delta m_o^2}{4k} \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \quad (14)$$

Again, adding a term proportional to the identity matrix, one can write this as

$$i \frac{d}{dt} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m_o^2}{4k} \cos 2\theta + \frac{\sqrt{2}}{2} G_f N_e & \frac{\Delta m_o^2}{4k} \sin 2\theta \\ \frac{\Delta m_o^2}{4k} \sin 2\theta & \frac{\Delta m_o^2}{4k} \cos 2\theta - \frac{\sqrt{2}}{2} G_f N_e \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}. \quad (15)$$

This can be written in a form identical to Eq. 9 by defining the quantities:

$$\begin{aligned} \frac{\Delta m_N^2}{4k} \cos 2\theta_N &= \frac{\Delta m_o^2}{4k} \cos 2\theta - \frac{\sqrt{2}}{2} G_F N_e \\ \frac{\Delta m_N^2}{4k} \sin 2\theta_N &= \frac{\Delta m_o^2}{4k} \sin 2\theta. \end{aligned} \quad (16)$$

Thus, the entire discussion of oscillations in matter is identical to that of oscillations in vacuum with the replacement of Δm_o^2 and $\sin 2\theta$ with Δm_N^2 and $\sin 2\theta_N$. Solving the above equation explicitly gives

$$\begin{aligned} \tan 2\theta_N &= \frac{1}{\cot 2\theta - \frac{2k}{\Delta m_o^2} \frac{\sqrt{2} G_F N_e}{\sin 2\theta}} \\ \Delta m_N^2 &= \Delta m_o^2 \frac{\sin 2\theta}{\sin 2\theta_N}. \end{aligned} \quad (17)$$

One can immediately see that if the electron density has the value such that the denominator of the equation for $\tan 2\theta_N$ vanishes, then one can have a huge mixing angle (45 degrees) even if the vacuum mixing angle is very small. This is the origin of the enormous enhancement one can have for oscillations in matter as opposed to vacuum. Since the electron density in the Sun varies from a large value at the core to a small value at the surface, it is possible that that critical electron density will be reached on the way out of the Sun.

Since the form of the propagator matrix is identical to that of Eq. 9 (with the replacements of Eq. 16), we have the flavor transition probability

$$P_{\text{transition}} = \sin^2 2\theta_N \sin^2 \left(1.27 \frac{\Delta m_N^2}{k} L \right) \quad (18)$$

The dependence of θ_N on the various parameters is shown in Fig. 3 for the case that the vacuum mixing angle $\theta = 0.1$. Note that the angle does, as stated in the previous

paragraph, reach 45 degrees for some value of the electron density. As the vacuum angle gets smaller, the width of the peak in Fig. 3 will decrease, but the height will always be at 45 degrees. Maximal mixing between flavor states occurs when $\theta_N = \pi/4$; this resonance condition is

$$\frac{\Delta m_o^2 \cos 2\theta}{2kN_e G_F \sqrt{2}} = 1. \quad (19)$$

It is interesting to note that far above resonance—as, for instance, in a medium of very high density—the mixing angle goes to zero. Notice also that if the vacuum mixing angle is negative, then the matter mixing angle always lies between zero and the vacuum value, tending towards zero as the density of the medium increases.

A detailed discussion of the transition of neutrinos in the Sun will be given in the next section, which deals with a non-constant density medium. The moon has a fairly constant density, and thus the results of this section will be sufficient to determine the isoprobability contours for neutrino flavor transition as they pass through the moon during a solar eclipse. For a lunar density of 3400 kg/m^3 (roughly that of the Earth's mantle), and for momenta on the order of that of solar neutrinos ($O(1) \text{ MeV}$), then the resonance condition for small vacuum mixing will be satisfied for $\Delta m_o^2 \simeq 10^{-6} \text{ eV}^2$, which is also, as we can see from Fig. 1, the range of masses allowed by solar neutrino experiments. Thus, resonant oscillations in the moon are not implausible. A detailed analysis of this effect will follow in Section 6.

There has also been much discussion[8] of the possibility of matter oscillations in the Earth (which also has a non-constant density). Electron neutrinos generated in the solar core can be converted to muon neutrinos in the Sun, and can then reconvert back to electron neutrinos in the Earth. Since only solar neutrinos detected at night have passed through the Earth, this is called the day/night effect, and the fact that some experiments do not yet see a significant day/night has been used to place some constraints in the

MSW plane. It is amusing to think that, if the day/night effect is observed, then the Sun (observed in electron neutrinos) could be brighter at night!!

5 The Matter Effect in a Non-Constant Density Medium

If the medium in which the neutrinos propagate does not have a constant density, then we must be much more careful. This case is especially relevant to the solar neutrino problem because the Sun's density varies strongly between the core and the photosphere. It is in fact roughly exponential, with a maximum density of 150,000 kg/m³ at the center.

The case of a slowly changing density profile is called adiabatic since the matter mass eigenstates will vary smoothly (i.e., without mixing between these states) into the vacuum mass eigenstates as the density goes to zero. If the density profile changes rapidly, then the mass eigenstates may be mixed at this point. The technical condition for adiabaticity is somewhat subtle; see the book of Bahcall[7] for details. We will first assume that the *matter mixing angle* changes slowly, which one can show, from the adiabaticity condition, is satisfied except at a resonance crossing (see Fig. 3).

The adiabatic evolution of a neutrino state is determined by a simple generalization of the results of the previous section:

$$i\frac{d}{dt}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = \begin{pmatrix}-\frac{\Delta m_o^2}{4k}\cos 2\theta + \frac{\sqrt{2}}{2}G_F N_e(t) & \frac{\Delta m_o^2}{4k}\sin 2\theta \\ \frac{\Delta m_o^2}{4k}\sin 2\theta & \frac{\Delta m_o^2}{4k}\cos 2\theta - \frac{\sqrt{2}}{2}G_F N_e(t)\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}. \quad (20)$$

This differs only in the time dependence of the electron number density. One can then define the time dependent mass and mixing angle as

$$\begin{aligned}\frac{\Delta m_N^2(t)}{4k}\cos 2\theta_N(t) &= \frac{\Delta m_o^2}{4k}\cos 2\theta - \frac{\sqrt{2}}{2}G_F N_e(t) \\ \frac{\Delta m_N^2(t)}{4k}\sin 2\theta_N(t) &= \frac{\Delta m_o^2}{4k}\sin 2\theta\end{aligned} \quad (21)$$

With these definitions, the expressions for $\nu_e(t)$ and $\nu_\mu(t)$ in terms of their values at $t = 0$ become identical to Eq. [16] with the time-dependence of θ_N and Δm_N^2 included. This

time-dependence, of course, can be trivially converted into a distance, since the neutrinos are travelling very close to the speed of light.

The non-adiabatic case is too complicated to deal with in detail here. We will summarize the useful results. The survival probability for a neutrino which propagates through a density resonance (whose existence is established by satisfying the resonance condition of Eq. [19] somewhere in the Sun) is

$$P_s = \sin^2 \theta + P_x \cos 2\theta \quad (22)$$

where P_x is the Landau-Zener level crossing probability. This factor gives the probability of a transition between the adiabatic states during resonance crossing. It is given by

$$P_x = \text{Exp}\left[-\frac{\pi \sin^2 2\theta}{2 \cos 2\theta} \frac{\Delta m_o^2}{2k} \frac{1}{\frac{1}{N_e} \frac{dN_e}{dt}\big|_{\text{res.}}}\right] \quad (23)$$

The logarithmic derivative in the denominator is evaluated at the time of the resonance crossing.

The probability of detecting a neutrino in its original flavor state after two resonance crossings is obtained by replacing P_x with $P_{1x}(1 - P_{2x}) + P_{2x}(1 - P_{1x})$. The generalization to many crossings is then simple.

Although straightforward, the detailed description of the evolution in the solar neutrino flux as the neutrinos leave the Sun must be done numerically, primarily because the neutrino spectrum emitted by the core in the first place (determined from the SSM) is fairly complicated, and all of the expressions are energy-dependent. Figure 4 shows the transition probability for neutrinos generated in the core of the Sun. The horizontal line at about $2 \times 10^{-4} \text{ eV}^2$ is due to the resonance crossing condition, Eq. [19]. Above this line, there is no resonance crossing in the Sun for the relevant range of neutrino energies, and the vacuum oscillations are the dominant effect. Below this line we must use the Landau-Zener expression above; the slope of the diagonal lines follow from the

form of P_x . Of course, no experiment is sensitive to the entire range of solar neutrinos; so each maps out a region in the plane. Combining all of the experiments gives the allowed region in Fig.1.

One must be cautioned about taking Fig. 1 too seriously; if one of the experiments is wrong, then the allowed regions can expand considerably, and thus when discussing the effects of the moon on solar neutrinos, we will consider the general region covered in Fig. 4. We now apply the MSW model to the possibility of seeing a change in the solar neutrino flux during a solar eclipse.

6 Neutrino Oscillations in the Moon

From the vantage point of a typical solar neutrino detector, the center of the Sun is covered for a few hours every decade during solar eclipses. Although total eclipses are rare, partial eclipses occur much more frequently, and one must remember that the Earth is transparent to neutrino detectors. Could neutrino oscillations in the moon be detected? Since there is not likely to be any systematic effect which “turns on” only during eclipses, one can integrate over a large number of eclipses. We will now analyze the possibility of observing a change in the solar neutrino rate during a solar eclipse.

Specifically, one calculates the percent change in the eclipsed signal relative to the uneclipsed signal for many points in the MSW plane, and plots the resulting contours in this plane. By “signal”, we mean the total number of expected solar neutrino events during the time of observation (on the order of an hour per eclipse). There are two possible effects that the moon can have on the signal. Neutrinos which survive their passage through the Sun as electron neutrinos can convert into muon or tau neutrinos in the moon, and neutrinos which transform on their way out of the Sun can reconvert in the moon. The latter effect will turn out to be more significant, thus the effect of the

moon is to *increase* the electron neutrino flux.

To find the expected uneclipsed signal for given values of Δm_o^2 and $\sin^2 2\theta$, we first calculate the event rate of the detector and then integrate this over the duration of the eclipse. The rate is obtained from the integrals

$$P = \sum_{i=e,\mu,\tau} \int \sigma_i(k) \phi_i(k) dk \quad (24)$$

where $\phi_i(k)$ is the spectral flux of neutrino type i arriving at the detector and σ_i is the cross-section for a neutrino interacting with a detector atom. For the reaction we are considering, $\nu_e + d \rightarrow p + p + e^-$ (the primary reaction for SNO), σ_μ and σ_τ both vanish; for the reaction $\nu_i + e^- \rightarrow \nu_i + e^-$, the values for σ_μ and σ_τ are about a factor of seven less than σ_e . The cross sections for the nuclear reaction are obtained from extrapolating known nuclear cross-sections to low energies. We used the nuclear cross-sections of Bahcall[7].

The spectral neutrino fluxes at the detector are obtained by propagating the initial ν_e signal through the sun and the vacuum using the techniques described in the previous three sections. We have

$$\phi_i(k) = \phi_e^o |\langle \nu_i(t) | \nu_e(0) \rangle|^2 \quad (25)$$

$$= \phi_e^o P_{e \rightarrow i}(k) \quad (26)$$

where $P_{e \rightarrow i}$ is the probability of an electron neutrino generated at the solar core being detected as an i -neutrino after propagating to the earth and ϕ_e^o is the initial neutrino flux from the standard solar model. For the deuteron break-up reaction at SNO, $i = e$ and thus this becomes, in more familiar terminology,

$$\phi_e(k) = \phi_e^o(k) P_s(k) \quad (27)$$

If there is no resonance crossing for a particular energy and combination of Δm_o^2 and θ on the way out of the sun, then P_s is obtained from Eq. 12, appropriately averaged

over the production position. If there is a resonance density crossing in the Sun, then P_s comes from Eq. 22.

The eclipsed signal is obtained in a similar manner, i.e. by integrating the rate over one eclipse, with the only difference being that the solar neutrino flux at the detector is modified by the intervention of the moon. This flux will also have some time-dependence due to the time dependence of the neutrino path length through the moon. The extreme non-adiabatic transition to the lunar medium (which may be thought of as a measurement) separates the states and allows one to ignore interference terms. Hence we use the classical probability result

$$\phi_e(k, t) = \phi_e^o(k)[P_{s,sun}(k)P_{s,moon}(k, t) + P_{t,sun}(k)P_{t,moon}(k, t)] \quad (28)$$

for the electron neutrino flux at the detector. $P_{(s,t)}$ specify the probability of survival (transition) for a flavor eigenstate propagating through the given medium. The probabilities for the moon are from Eq. 18 with the electron density appropriate to the moon (about 10^{30}m^{-3}). In this case

$$L(t) = 2(R^2 - b^2 - (vt)^2)^{1/2} \quad (29)$$

Here, R is the radius of the moon, v is the velocity of the moon relative to the sun as seen from the earth (about 1 km/sec) and b is the impact parameter of the eclipse (zero for a total eclipse). We have set $t = 0$ at the eclipse maximum. Since the density of the moon is (to an accuracy of ten percent or so) constant, Eq. 18 will be valid for all energies and regions of the parameter space; i.e. there is no “resonance crossing” as for the Sun. There is, however, a resonance effect due to the length of oscillations in the lunar medium becoming comparable to the lunar diameter. This enhances flavor oscillations strongly in the vicinity of $\Delta m_o^2 = 5 \times 10^{-6}\text{eV}^2$, as shown in Fig. 5, in which we plot the flavor transition probability for the moon in the same manner as done for the Sun in Fig. 4. Note that this is a region of parameter space which is interesting to us (see Fig 1).

7 Results and Conclusion

We calculated the total percentage change in the electron neutrino signal which would be observed through $\nu_e + d \rightarrow p + p + e^-$ at the Sudbury Neutrino Observatory, for 900 points in the MSW plane. Fig. 6 is the result. The figure may be intuitively understood by imagining the “multiplication” of Figs. 4 and 5 (remembering that these figures are for a specific neutrino energy, whereas for Fig. 6 we have integrated over the spectrum). For example, note the large peak at $\sin^2 2\theta \simeq 0.4$, $\Delta m_o^2 \simeq 7 \times 10^{-6} \text{eV}^2$ for which the flux is increased by a factor of 7! As one can see from Fig. 4, however, for these parameters over 90 percent of the electron neutrinos emitted by the core of the Sun are converted within the Sun; the factor of 7 simply means that some number of these are converted back. This region is already excluded, as can be seen in Fig. 1. Comparing with Fig. 1, we can see that for the small angle MSW solution, there is no measureable effect from the moon. However, for the large angle solution, there is an enhancement in the signal which ranges from 50 to 100 percent.

The calculation is presented with impact parameter $b = 0$. It was repeated with impact parameters $0.25R$, $0.5R$ and $0.75R$, and the dependence of Fig. 6 was found to be weak. This is not surprising due to the square-root dependence of the maximum neutrino path length in the moon on the impact parameter.

With the current generation of detectors, statistically significant data is an impossibility for this measurement. However, the coming generation detectors SNO, Super-Kamiokande, Icarus and Borexino will all have rates on the order of one event per hour[11]. Adding the data from multiple eclipses will provide a larger sample. Over the next 15 years, there will be roughly five hours during which the center of the Sun is covered at all of these sites[12]. At Borexino, for example, one would expect about 10 events during these time periods (and many more in the next round of detectors).

From Fig. 6, one can see that there is a region in the MSW plane in which a statistically significant detection of the moon’s solar neutrino shadow is possible.

In this article, we have shown how neutrino oscillations can explain the observed deficit in solar neutrinos. Even with a very small mixing angle, oscillations can be resonantly enhanced as the neutrinos travel from the core of the Sun to the surface. In addition to a pedagogic review, we have examined the possibility that the solar neutrinos could undergo further transformation if they pass through the moon during a solar eclipse. There is a region in the MSW plane in which a change in the neutrino flux during an eclipse could be observed in the next generation of solar neutrino detectors.

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References

- [1] R. Davis, et al., “Search for Neutrinos from the Sun”, *Phys. Rev. Lett.* **20**, 1205-1209 (1968).
- [2] K.S. Hirata, et al., “Observation of 8B Solar Neutrinos in the Kamiokande-II detector”, *Phys. Rev. Lett.* **62**, 16-19 (1989).
- [3] A.I. Abasov, et al., “Search for Neutrinos from the Sun using the reaction $^{71}Ga(\nu_e, e^-)^{71}Ge$ ”, *Phys. Rev. Lett.* **67**, 3332-3335 (1991).
- [4] P. Anselmann, et al., “Implications of the GALLEX determination of the Solar Neutrino Flux”, *Phys. Lett.* **B285**, 390-397 (1992).
- [5] S.P. Mikheyev and A. Yu. Smirnov, “Resonant enhancement of oscillations in matter and solar neutrino spectroscopy”, *Sov. J. Nucl. Phys.*, **42** 913-917 (1986).

- [6] L. Wolfenstein, “Neutrino Oscillations in Matter”, *Phys. Rev.* **D17**, 2369-2374 (1978)
- [7] J.N. Bahcall, *Neutrino Astrophysics* (Cambridge Univ. Press, Cambridge, 1989).
- [8] N. Hata and P. Langacker, “Earth Effect in MSW Analysis of Solar Neutrino Experiments”, *Phys. Rev.* **D48**, 2937-2940 (1993).
- [9] R. Bernstein and S. Parke, “Terrestrial Long Baseline Neutrino Oscillation Experiments”, *Phys. Rev.* **D44**, 2069-2077 (1991).
- [10] H. Bethe, “Possible Explanation of the Solar Neutrino Puzzle”, *Phys. Rev. Lett.* **56**, 1305-1308 (1986).
- [11] J.-P. Revol, “Future Neutrino Oscillation Experiments”, Proceedings of the Rencontres de Physique de la Valee d’Aosta 1993, (March 1993).
- [12] This was obtained by using “Dance of the Planets” to find the times of all eclipses (partial and total) over the next fifteen years, and watching the Sun (through a transparent earth, if necessary) from each of the solar neutrino detector sites. The center of the Sun is covered for roughly five hours from each site during that period.

Figure Captions

Fig. 1 - The regions of the $\Delta m^2 - \sin^2 2\theta$ plane allowed by combining all of the solar neutrino experiments to date, assuming that solar neutrino oscillations are the reason for the observed deficit. This figure is Fig. 3 from Ref. 8; we thank Paul Langacker for providing us with a copy of the figure.

Fig. 2- Interactions of neutrinos with electrons. All neutrinos interact with electrons via Z^0 -exchange, but only electron neutrinos can interact via W^+ exchange.

Fig. 3- The matter mixing angle, assuming a vacuum mixing angle of 0.1, as a function of β , where $\beta = \frac{2\sqrt{2}G_F N_e}{\sin 2\theta} \frac{k}{\Delta m_o^2}$. We have plotted $|\theta_N|$ since only $\sin^2 2\theta_N$ is relevant; θ_N is negative to the right of the peak.

Fig. 4- Transition probability for electron neutrinos travelling from the core of the Sun to the surface. Contour intervals are 10 percent. We have taken the neutrino energy to be 7 MeV for this plot; different energies will yield similar curves.

Fig. 5- Transition probability for electron neutrinos travelling through the moon. As in Fig. 4, the neutrino energy is taken to be 7 MeV. Contour intervals are 10 percent.

Fig. 6- Total change in the neutrino flux observed at SNO during a solar eclipse. We have plotted the fractional change (so that 100 percent corresponds to a factor of two increase in the flux). The contours, starting with the outer contour and working inward, are 5, 50, 100, 200, 400 and 600 percent, respectively.

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